DESIGN MANUAL FOR 4EQ STRUCTURAL BAR GFRP REINFORCEMENT

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ICC-ES AC 454

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1. INTRODUCTION

1.1. PURPOSE

The objective of the design manual is to satisfy the design requirement of the ESR (evaluation service report) No. 4664 in recognition of the Acceptance Criteria AC 454 International Code Council Evaluation Service (ICC-ES) Acceptance Criteria for Fiber-Reinforced Polymer (FRP) Bars for Internal Reinforcement of Concrete Members (AC 454), Section 8.0: for the use of 4EQ Structural Bar® for uses as internal reinforcement in concrete members.

This document contains the experimental and design, material specifications, design equations per ACI 440.1R, installation instructions, reference to design tools for the licensed professional and design examples for reference purposes.

1.2. DESIGN MANUAL HOLDER INFORMATION

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2. 4EQ STRUCTURAL BAR®

2.1. 4EQ STRUCTURAL BAR® DESCRIPTION

4EQ Structural Bar® is a high modulus glass fiber reinforced polymer (GFRP) rebar composed of highly resistant glass fibers embedded in a vinyl ester resin. The GFRP bars have a continuously profiled surface. 4EQ Structural Bar®, include straight bars, designed for use as internal reinforcement of concrete for the construction of structures and elements in residential, commercial, industrial, road and civil engineering.

2.2. 4EQ STRUCTURAL BAR® EXPERIMENTAL PROPERTIES

The following reported experimental properties of the 4EQ Structural Bar® are absolute values based on experimental test specimens. These values do not include safety factors; refer to Section 4 of this document for design properties. Safety factors must be applied to values in accordance with American Concrete Institute Design Guide ACI 440.1R-15.

TABLE 1—NOMINAL PROPERTIES AND MINIMUM GUARANTEED TENSILE LOAD FOR 4EQ STRUCTURAL BAR® GLASS FRP STRAIGHT BAR

MEASURED		NOMINAL			
BAR AREA RANGE (in^2)	MINIMUM GUARANTEED TENSILE LOAD (kips)	BAR DESIGNATION	BAR DIAMETER (in)	BAR AREA (in^2)	TENSILE STRENGTH (ksi)
0.104 TO 0.161	18.1	3	0.450	0.160	181.1

For SI: 1.0 in^2 = 645 mm^2; 1.0 in = 25.4 mm, 1.0 kip = 4.45 kN
* Based on Nominal bar area.

3. INSTALLATION OF 4EQ STRUCTURAL BAR®

3.1. INSTALLATION INSTRUCTION

For flexural and shear members, there is no restriction for the shape of concrete member cross-section (e.g., rectangular, T-shape, L-shape). For flexural reinforcement, the use of multiple 4EQ Structural Bar® layers and bar bundling is permitted. For multiple bar layers, the relevant provisions for steel reinforcing bar in ACI 318 also apply to 4EQ Structural Bar®. Because GFRP materials have no plastic region, the stress in each reinforcement layer varies depending on its distance from the neutral axis. Thus, the analysis of the flexural capacity shall be based on a strain-compatibility approach. For bundled bars, all relevant provisions of ACI 318 apply.

Design and installation must be in accordance with the published ICC-ES report, the approved quality documentation, this Design Manual, and the IBC and IRC. Complete construction documents, including plans and calculations verifying compliance with the evaluation report must be submitted to the code official for each project at the time of permit application. The construction documents must be prepared and sealed by a registered design professional where required by the statutes of the jurisdiction in which the project is to be constructed.

Fire-resistance rating of 4EQ Structural Bar® reinforced concrete assembly is outside the scope of the evaluation report, and concrete assemblies with 4EQ Structural Bar® are limited to type VB construction under the IBC.

Special inspection as required by Table 1705.3 of the IBC for steel-reinforced concrete construction, is also applicable to 4EQ Structural Bar® reinforced concrete construction.

3.2. ENVIRONMENTAL CONSIDERATIONS

Ambient temperature at the time of installation shall be 55° F to 100° F (13° C to 38° C). 4EQ Structural Bar® shall be stored above the surface of the ground on platforms, skids, or other supports as close as possible to the point of placement. If stored outdoors, the 4EQ Structural Bar® reinforcing bars shall be covered with opaque plastic or other types of cover that will protect the bars from ultraviolet rays.

3.3. PRODUCT IDENTIFICATION

All 4EQ Structural Bar® products are labeled in accordance with the approved quality control documentation including the manufacturer name and address, product name and evaluation report number. Prior to installation, verify that GFRP is legitimate.

4. DESIGN PROVISIONS

This Design Manual is applicable to non-prestressed FRP bars that are solid and have circular cross sections. FRP bars under this DM are used as:

- Flexural reinforcement in structural concrete members such as beams, shallow foundations and one-way or two-way slabs,
- Shear reinforcement for flexural members.
- Longitudinal reinforcement in columns or walls.

4.1. DESIGN MATERIAL PROPERTIES

The provisions listed are derived from Chapter 6 of ACI 440.1R-15 and ACI 318-. Equations (A-1) through (A-3) give the tensile properties that shall be used in all design equations. The design tensile strength shall be determined by:

$$f_{fu} = C_E \, f_{fu}* \qquad \text{(A-1)}$$

where:
f_{fu} = design tensile strength of 4EQ Structural Bar®, considering reductions for service environment, psi
C_E = environmental reduction factor equal to 0.7 and 0.8 for concrete exposed to earth and weather and not exposed, respectively (See Table 6.2 of ACI 440.1R-15).
f_{fu}* = nominal tensile strength of a 4EQ Structural Bar® defined as the guaranteed tensile load divided by the nominal cross-sectional area, psi = 181.1 ksi (see Table 1)

The design rupture strain shall be determined as:

$$\varepsilon_{fu} = C_E \, \varepsilon_{fu}* \qquad \text{(A-2)}$$

where:
ε_{fu} = design rupture strain of FRP reinforcement
ε_{fu}* = calculated ultimate rupture strain of 4EQ Structural Bar® reinforcement defined as the guaranteed tensile load divided by the product of mean elastic modulus and nominal cross-sectional area = 0.019

The design modulus of elasticity for 4EQ STRUCTURAL BAR® GLASS FRP STRAIGHT BAR:

E = 9,427 ksi (65 GPa)

Tensile strength of solid FRP bars at bends—The design tensile strength of solid FRP bars at a bend shall be determined as:

$$f_{fb} = \left(0.05\frac{r_b}{d_b} + 0.3\right) f_{fu} \leq f_{fu} \qquad \text{(A-3)}$$

where:

f_{fb} = design tensile strength of the bend of FRP bar, psi
r_b = radius of the bend, in.
d_b = nominal diameter of reinforcing bar, in.
f_{fu} = design tensile strength of FRP, considering reductions for service environment, psi

For 4EQ STRUCTURAL BAR®

$$f_{fb} = (0.05(3) + 0.3)181.1\ ksi = 81.5\ ksi$$

Per ACI 440.1R Section 8.3 the minimum ratio of r_b / d_b = 3 is recommended.

The value of f_{fb} as obtained in Eq. (A-3) shall not be greater than the mean strength of the bend obtained experimentally.

At columns, if factored axial compression $P_u > 0.10\ f_c'\ A_g$, the tensile design strain of the longitudinal GFRP bars shall be limited to 0.01. The corresponding design strength, f_{fd}, shall then be calculated as:

$$f_{fd} = Min\ (f_{fu}, 0.01E_f) \qquad \text{(A-4)}$$

Attaining the full tensile capacity of the GFRP bars requires a degree of curvature which may be either unattainable or unacceptable for columns, and an upper limit on ultimate tensile strain is therefore imposed. Thus, for design purposes when factored axial compression Pu > 0.10 f_c' A_g, the ultimate tensile design strain may not exceed a fixed limit of 1 percent.

4.2. FRP CREEP RUPTURE AND FATIGUE

To account for potential failure of the FRP reinforcement due to creep rupture or fatigue, FRP stress at service shall be limited to 30 percent of the guaranteed and nominal tensile strength, respectively.

4.3. FRP BAR ARRANGEMENT

For FLEXURAL reinforcement, the use of multiple bar layers and bar bundling is permitted. For multiple bar layers, the relevant provisions for steel reinforcing bar in ACI 318 also apply to FRP bars. Because FRP materials have no plastic region, the stress in each reinforcement layer varies depending on its distance from the neutral axis. Thus, the analysis of the flexural capacity shall be based on a strain-compatibility approach. For bundled bars, all relevant provisions of ACI 318 apply.

4.4. FLEXURAL AND SHEAR REINFORCEMENT

4.4.1. Flexural reinforcement in structural concrete members such as beams, shallow foundations and one-way or two-way slabs must follow the design provisions as given in Chapter 7 of ACI 440.1R-15.

4.4.2. Shear reinforcement for flexural members is outside the scope of this report. Refer to Chapter 8 of ACI 440.1R-15 for design provisions

4.4.3. Design examples given in ACI 440.1R can be used for guidance.

4.5. LONGITUDINAL REINFORCEMENT IN COLUMNS

This chapter addresses longitudinal reinforcement in columns or walls and applies to the design of non-prestressed columns reinforced with GFRP bars, including reinforced concrete pedestals.

4.5.1. DIMENSIONAL LIMITS

- For columns with a square, octagonal, or other shaped cross section, it shall be permitted to base gross area considered, required reinforcement, and design strength on a circular section with a diameter equal to the least lateral dimension of the actual shape.
- For columns built monolithically with a concrete wall, the outer limits of the effective cross section of the column shall not be taken greater than 1.5 in. outside the transverse reinforcement.
- For columns with two or more interlocking spirals, outer limits of the effective cross section shall be taken at a distance outside the spirals equal to the minimum required concrete cover.
- If a reduced effective area is considered according to the first three bullet points, structural analysis and design of other parts of the structure that interact with the column shall be based on the actual cross section.
- Bent shapes, continuous closed stirrups, and ties (hoops) that are used to reinforce concrete structural members are outside the scope of this manual. Refer to ACI 440.1R-15 for all reinforcement requirements and design requirements.

4.5.2. STRAIN LIMITS

If factored axial compression $P_u > 0.10f_c'A_g$, the tensile design strain of the longitudinal GFRP bars shall be limited to 0.01. The corresponding design strength, f_{fd}, shall then be calculated as:

$$f_{fd} = Min\left(f_{fu}, 0.01E_f\right) \quad \text{(A-4)}$$

4.5.3. REQUIRED STRENGTH

Provisions for minimum spacing as specified in ACI 318. These provisions apply to crossties as well as ties.

Maximum axial strength

Transverse Reinforcement	$P_{n,max}$
Ties conforming to ACI 318	0.80 P_o
Spirals conforming to ACI 318	0.85 P_o

The minimum amount of shear reinforcement when Vu exceeds $\phi V_c/2$ shall be the greater of:

$$A_{fv,min} = 0.75\sqrt{f_c'}\frac{b_w s}{f_{fv}} \quad \text{(A-5)}$$

$$A_{fv,min} = \frac{50 b_w s}{f_{fv}} \quad \text{(A-6)}$$

Maximum spacing of shear reinforcement shall be the lesser of d/4 or 12 in. The maximum spacing of vertical stirrups shall be the smaller of d/2 or 24 in. This limit ensures that each shear crack is intercepted by at least one stirrup. Additionally, a minimum tail length of $12d_b$ is specified for open stirrups.

If the bar force due to factored loads is compressive, compression lap splices shall be permitted. Compression splices shall be designed in accordance with ACI 318 assuming $f_f = 0.25\ f_{fu}$. It shall be permitted to decrease the compression lap splice length in accordance with (a) or (b), but the lap splice length shall be at least 12 in.

4.6. LONGITUDINAL REINFORCEMENT IN WALLS

This chapter applies to the design of non-prestressed walls including (a) through (c):

(a) Cast-in-place
(b) Precast in-plant
(c) Precast on-site including tilt-up

4.6.1. DESIGN LIMITS

Minimum wall thickness: Minimum wall thicknesses must be in accordance with Table below. Thinner walls are permitted if adequate strength and stability can be demonstrated by structural analysis.

Minimum wall thicknesses shall be in accordance with Table 4.6.1. Thinner walls are 80 permitted if adequate strength and stability can be demonstrated by structural analysis.

TABLE 4.6.1 – MINIMUM WALL THICKNESS, h

WALL TYPE	MINIMUM WALL THICKNESS		
BEARING		5.5 IN.	(a)

	GREATER OF:	1/24 THE LESSER OF UNSUPPORTED LENGTH AND UNSUPPORT HEIGHT	(b)
NON-BEARING	GREATER OF:	4 IN.	(c)
		1/30 THE LESSER OF UNSUPPORTED LENGTH AND UNSUPPORT HEIGHT	(d)
EXTERIOR BASEMENT AND FOUNDATION [1]	7.5 IN.		(e)

[1] Only applies to walls designed in accordance with the simplified design method of ACI 318.

4.6.2. REQUIRED STRENGTH

Provision for required strength of wall shall follow the ACI 318 specification with the following modifications:

- The effects of fire shall be considered in design.
- If the resultant of all factored loads is located within the middle third of the thickness of a solid wall with a rectangular cross section, P_n shall be permitted to be calculated by:

$$P_n = 0.45 f_c{}' A_g \left[1 - \left(\frac{k l_c}{32h}\right)^2\right] \qquad \text{(A-7)}$$

- V_n at any horizontal section shall not exceed 0.2 f_c'hd
- V_c shall be calculated per ACI 318 with the term b_w replaced by h.
- V_f shall be provided by transverse shear reinforcement and shall be calculated by:

$$V_f = \frac{A_{fv} f_{fv} d}{s} \qquad \text{(A-8)}$$

- The value of f_{fu} used to calculate V_f shall be the smaller of $0.005E_f$ or f_{fb} where f_{fb} is defined ACI 318.
- Provisions for minimum reinforcement spacing as specified in ACI 318. These provisions apply to vertical and horizontal reinforcement.
- If in-plane $Vu \leq 0.5\phi Vc$, minimum ρ_ℓ and minimum ρ_t shall be 0.0036. These limits need not be satisfied if adequate strength and stability can be demonstrated by structural analysis.
- If in-plane $Vu \geq 0.5\phi Vc$, (a) and (b) shall be satisfied:
  (a) ρ_ℓ shall be at least 0.0055 but need not exceed ρ_t required for strength by ACI 318
  (b) ρ_t shall be at least 0.0055.

- Spacing s of longitudinal bars in cast-in-place walls shall not exceed the lesser of 3h and 18 in. If shear reinforcement is required for in-plane strength, spacing of longitudinal reinforcement shall not exceed $\ell_w/3$.

- In addition to the minimum reinforcement required by ACI 318 Section 11.6, at least four No. 5 bars in walls having two layers of reinforcement in both directions and two No. 5 bars in walls having a single layer of reinforcement in both directions shall be provided around window, door, and similarly sized openings. In lieu of more detailed analysis that shows lower bar stresses can be considered under factored loads, such bars shall be anchored to develop f_{fu} in tension at the corners of the openings. An additional two No. 5 bars in walls having two layers of reinforcement in both directions and one No. 5 bar in walls having a single layer of reinforcement in both directions shall be placed diagonally at each corner. Diagonal bars shall have a minimum anchorage length of 24 in. from the corner to either end of the bar.

- Reinforcement supporting cyclic loading, bent shapes, continuous closed stirrups, and ties (hoops) that are used to reinforce concrete structural members are outside the scope of this manual. Refer to ACI 440.1R-15 for all reinforcement requirements and design requirements.

4.7. DESIGN NOMENCLATURE

A_f	=	area of FRP reinforcement, in^2
$A_{f,min}$	=	minimum area of FRP reinforcement needed to prevent failure of flexural members upon cracking, in^2
A_{fv}	=	amount of FRP shear reinforcement within spacing s, in^2
$A_{fv,min}$	=	minimum amount of FRP shear reinforcement within spacing s, in^2
A_g	=	area of gross concrete section (b * d), in^2
a	=	depth of equivalent rectangular stress block, in.
b	=	width of rectangular cross-section, in.
b_o	=	perimeter of critical section for slabs and footings, in.
b_w	=	width of the web, in.
C	=	spacing or cover dimension, in.
C_E	=	environmental reduction factor for various exposure conditions
c	=	distance from extreme compression fiber to the neutral axis, in.
c_b	=	distance from extreme compression fiber to neutral axis at balanced strain condition, in.
cc	=	clear concrete cover, in.
d	=	distance from extreme compression fiber to centroid of tension reinforcement, in.
d_{agg}	=	diameter of concrete aggregate, in.
d_b	=	diameter of reinforcing bar, in.
d_c	=	thickness of concrete cover measured from extreme tension fiber to center of bar or wire location closest thereto, in.
E_f	=	design or guaranteed modulus of elasticity of FRP defined as mean modulus of sample of test specimens ($E_f = E_{f,ave}$), psi
$E_{f,ave}$	=	average modulus of elasticity of FRP, psi
E_s	=	modulus of elasticity of steel, psi
f'c	=	specified compressive strength of concrete, psi
√f'c	=	square root of specified compressive strength of concrete, psi
f_f	=	stress in FRP reinforcement in tension, psi
f_{fb}	=	strength of bent portion of FRP bar, psi
f_{fe}	=	bar stress that can be developed for embedment length le, psi
f_{fr}	=	required bar stress, psi
$f_{f,s}$	=	stress level induced in FRP by sustained loads, psi
f_{fu}	=	design tensile strength of FRP, considering reductions for service environment, psi
f_{fu}*	=	guaranteed tensile strength of FRP bar, defined as mean tensile strength of sample of test specimens minus three times standard deviation ($f_{fu}^* = f_{fu,ave} - 3\sigma$), psi

f_{fv} = tensile strength of FRP for shear design, taken as smallest of design tensile strength f_{fu}, strength of bent portion of FRP stirrups f_{fb}, or stress corresponding to $0.004E_f$, psi

$f_{u,ave}$ = mean tensile strength of sample of test specimens, psi

I_{cr} = moment of inertia of transformed cracked section, in^4

I_e = effective moment of inertia, in^4

I_g = gross moment of inertia, in^4

k = ratio of depth of neutral axis to reinforcement depth

k_b = bond-dependent coefficient

l_a = additional embedment length at support or at point of inflection, in.

l_{bhf} = basic development length of FRP standard hook in tension, in.

l_d = development length, in.

l_e = embedded length of reinforcing bar, in.

l_{thf} = length of tail beyond hook in FRP bar, in.

M_a = maximum moment in member at stage deflection is computed, lb-in.

M_{cr} = cracking moment, lb-in.

M_n = nominal moment capacity, lb-in.

M_s = moment due to sustained load, lb-in.

M_u = factored moment at section, lb-in.

n_f = ratio of modulus of elasticity of FRP bars to modulus of elasticity of concrete

r_b = internal radius of bend in FRP reinforcement, in.

s = stirrup spacing or pitch of continuous spirals, and longitudinal FRP bar spacing, in.

T_g = glass transition temperature, °F

V_c = nominal shear strength provided by concrete, lb

V_f = shear resistance provided by FRP stirrups, lb

V_n = nominal shear strength at section, lb

V_u = factored shear force at section, lb

w = maximum crack width, in.

α = angle of inclination of stirrups or spirals and top bar modification factor

β = ratio of distance from neutral axis to extreme tension fiber to distance from neutral axis to center of tensile reinforcement (Section 8.3.1)

β_1 = factor taken as 0.85 for concrete strength f_c' up to and including 4000 psi. For strength above 4000 psi, this factor is reduced continuously at a rate of 0.05 per each 1000 psi of strength in excess of 4000 psi, but is not taken less than 0.65

β_d	=	reduction coefficient used in calculating deflection
$\Delta_{(cp+sh)}$	=	additional deflection due to creep and shrinkage under sustained loads, in.
$(\Delta_i)_{sus}$	=	immediate deflection due to sustained loads, in.
ε_{cu}	=	ultimate strain in concrete
ε_{fu}	=	design rupture strain of FRP reinforcement
ε_{fu}^*	=	guaranteed rupture strain of FRP reinforcement defined as the mean tensile strain at failure of sample of test specimens minus three times standard deviation ($\varepsilon_{fu}^* = \varepsilon_{u,ave} - 3\sigma$), in./in.
$\varepsilon_{u,ave}$	=	mean tensile strain at rupture of sample of test specimens
ξ	=	time-dependent factor for sustained load
ρ_f	=	FRP reinforcement ratio
$\rho_{f'c}$	=	ratio of FRP compression reinforcement
ρ_{fb}	=	FRP reinforcement ratio producing balanced strain conditions
$\rho_{f,ts}$	=	reinforcement ratio for temperature and shrinkage FRP reinforcement
σ	=	standard deviation
ϕ	=	strength reduction factor

4.8. DESIGN EQUATIONS

Equation	No.	Reference
$f_{fu} = C_E f_{fu} *$	(A-1)	ACI 440.1R EQ (6.2a)
$\varepsilon_{fu} = C_E \varepsilon_{fu} *$	(A-2)	ACI 440.1R EQ (6.2b)
$f_{fb} = \left(0.05 \frac{r_b}{d_b} + 0.3\right) f_{fu} \leq f_{fu}$	(A-3)	ACI 440.1R EQ (6.2.1)
$f_{fd} = Min\,(f_{fu}, 0.01E_f)$	(A-4)	
$A_{fv,min} = 0.75\sqrt{f_c{}'}\frac{b_w s}{f_{fv}}$	(A-5)	
$A_{fv,min} = \frac{50 b_w s}{f_{fv}}$	(A-6)	
$P_n = 0.45 f_c{}' A_g \left[1 - \left(\frac{k l_c}{32h}\right)^2\right]$	(A-7)	
$V_f = \frac{A_{fv} f_{fv} d}{s}$	(A-8)	
$\phi Mn \geq Mu$		ACI 440.1R EQ (7.2)
$\rho_f = \frac{A_f}{bd}$		ACI 440.1R EQ (7.2.1a)
$\rho_{fb} = 0.85\beta_1 \frac{f_c{}'}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}$		ACI 440.1R EQ (7.2.1b)
$M_n = A_f f_f \left(d - \frac{a}{2}\right)$		ACI 440.1R EQ (7.2.1a)
$a = \frac{A_f f_f}{0.85 f_c{}' b}$		ACI 440.1R EQ (7.2.1b)
$f_f = E_f \varepsilon_{cu} \frac{\beta_1 d - a}{a} \leq f_{fu}$		ACI 440.1R EQ (7.2.1c)
$M_n = A_f f_{fu} \left(d - \frac{\beta_1 c_b}{2}\right)$		ACI 440.1R EQ (7.2.1g)
$c_b = \left(\frac{\epsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}}\right) d$		ACI 440.1R EQ (7.2.1h)
$\varphi = \begin{cases} 0.55 \; for \; \rho_f \leq \rho_{fb} \\ 0.3 + 0.25 \frac{\rho_f}{\rho_{fb}} \; for \; \rho_{fb} < \rho_f < 1.4\rho_{fb} \\ 0.65 \; for \; \rho_f \geq 1.4\rho_{fb} \end{cases}$		ACI 440.1R EQ (7.2.3)
$A_{f,min} = 4.9 \frac{\sqrt{f_c{}'}}{f_{fu}} b_w d \geq \frac{300}{f_{fu}} b_w d$		ACI 440.1R EQ (7.2.4)

$V_C = 5\sqrt{f_c{'}}\, b_w kd$ ACI 440.1R EQ (8.2a)

$f_{fv} = 0.004E_f \le f_{fb}$ ACI 440.1R EQ (8.2d)

$\frac{A_{fv}}{s} = \frac{(V_u - \emptyset V_c)}{\emptyset f_{fv} d}$ ACI 440.1R EQ (8.2e)

$V_f = \frac{A_{fv} f_{fv} d}{s}(\sin\alpha + \cos\alpha)$ ACI 440.1R EQ (8.2f)

$V_f = \frac{A_{fv} f_{fv} d}{s}(\sin\alpha)$ ACI 440.1R EQ (8.2g)

$A_{fv,min} = \frac{50 b_w s}{f_{fv}}$ ACI 440.1R EQ (8.2.2)

$Vc = 10\sqrt{f_c{'}}\, b_o kd$ ACI 440.1R EQ (8.4a)

$s_{max} = 1.15\frac{E_f w}{f_f k_b} - 2.5c_c \le 0.92\frac{E_f w}{f_f k_b}$ ACI 440.1R EQ (7.3.1a)

$d_c \le \frac{E_f w}{2 f_1 \beta k_b}$ ACI 440.1R EQ (7.3.1b)

$I_{cr} = \frac{bd^3}{3}k^3 + n_f A_f d^2 (1-k)^2$ ACI 440.1R EQ (7.3.2.2a)

$k = \sqrt{2\rho_f n_f + \left(\rho_f n_f\right)^2} - \rho_f n_f$ ACI 440.1R EQ (7.3.2.2b)

$I_e = \frac{I_{cr}}{1-\gamma\left(\frac{M_{cr}}{M_a}\right)^2\left[1-\frac{I_{cr}}{I_g}\right]} \le I_g$ ACI 440.1R EQ (7.3.2.2c)

$\Delta_{(cp+sh)} = 0.6\,\xi(\Delta_i)_{sus}$ ACI 440.1R EQ (7.4.1)

$f_{f,s} = M_s \frac{n_f d(1-k)}{I_{cr}}$ ACI 440.1R EQ (7.3.2.3a)

$\rho_{f,ts} = 0.0018\frac{60{,}000}{f_{fu}}\frac{E_s}{E_f}$ ACI 440.1R EQ (9.1)

$f_{fe} = \frac{\sqrt{f_c{'}}}{\alpha}\left(13.6\frac{l_e}{d_b} + \frac{C}{d_b}\frac{l_e}{d_b} + 340\right) \le f_{fu}$ ACI 440.1R EQ (10.1c)

$$l_{bhf} = \begin{cases} 2000\frac{d_b}{\sqrt{f_c{'}}}\ for\ f_{fu} \le 75{,}000\ psi \\ \frac{f_{fu}}{37.5}\frac{d_b}{\sqrt{f_c{'}}}\ for\ 75{,}000 < f_{fu} < 150{,}000\ psi \\ 4000\frac{d_b}{\sqrt{f_c{'}}}\ for\ f_{fu} \ge 150{,}000\ psi \end{cases}$$ ACI 440.1R EQ (10.2b)

$$l_d = \frac{d_b\left(\frac{f_{fr}}{\sqrt{f_c{}'}} - 340\right)\varphi_t}{13.6 + \frac{c_b}{d_b}} d_b$$ ACI 440.1R EQ (10.3a)

$$l_d \leq \frac{\varphi M_n}{V_u} + l_a$$ ACI 440.1R EQ (10.3b)

5. DESIGN EXAMPLES

5.1. BEAM REINFORCEMENT DESIGN - FLEXURE

Calculate moment strength for rectangular beam based on static equilibrium using the equivalent rectangular concrete stress distribution.

Assume:
b = 10”
d = 16”
f_c' = 4000 psi
Interior exposure conditions and neglect compression reinforcement.
(6) 4EQ bars with 1.5” clear cover and 4EQ U-stirrups

4EQ MST bar properties:
d_b = 0.45 in.
$A_{f,bar}$ = 0.16 in^2
E_f = 9427 ksi

Design material properties:
C_E = 0.8 (interior exposure)
$f_{fu} = C_E f_{fu}^*$ = (0.8) (181.1) = 144.9 ksi

1. Determine the strength reduction factor.

   d = 16 – 1.5 – 0.45 – (0.45/2) = 13.83 in

   A_f = (6) (0.16 in^2) = 0.96 in^2

   $$\rho_f = \frac{A_f}{bd} = \frac{0.96}{(10)(13.83)} = 0.0047$$

   $$\rho_{fb} = 0.85\beta_1 \frac{f_c'}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}$$

   $$E_f \varepsilon_{cu} = 9427\ ksi(0.003) = 28.3 \text{ ksi}$$

   $$\rho_{fb} = 0.85(0.85)\frac{(4)}{(144.9)}\frac{(28.3)}{(28.3 + 144.9)} = 0.0033$$

   $$\frac{\rho_f}{\rho_{fb}} = \frac{0.0047}{0.0033} = 1.42$$

   Because $\rho_f \geq 1.4\rho_{fb}$, the section is compression-controlled therefore ϕ = 0.65

2. Determine stress in tensile reinforcement at ultimate conditions.

   $$f_f = \sqrt{\frac{(E_f \varepsilon_{cu})^2}{4} + \frac{0.85\beta_1 f_c'}{\rho_f} E_f \varepsilon_{cu}} - 0.5 E_f \varepsilon_{cu} \leq f_{fu}$$

$$f_f = \sqrt{\frac{(28.3)^2}{4} + \frac{0.85(0.85)(4)}{(0.0047)}(28.3)} - 0.5(28.3) \leq 144.9\ ksi$$

$$f_f = 118.52\ ksi$$

3. Determine nominal flexural strength M_n and design flexural strength ϕM_n.

$$a = \frac{A_f f_f}{0.85 f_c' b} = \frac{(0.96)(118.52)}{0.85(4)(10)} = 3.35\ in$$

$$M_n = A_f f_f\left(d - \frac{a}{2}\right) = (0.96)(118.52)\left(13.83 - \frac{3.35}{2}\right) = 1{,}383\ in - kip$$

$$= 115.2\ ft - kip$$

$$\emptyset M_n = (0.65)(115.2) = 74.9\ ft - kip$$

4. The minimum reinforcement provisions do not apply because the section is not tension-controlled.

   Note: While the general procedure and principles used in this example are applicable for an FRP-reinforced beam of any cross-sectional shape, the specific equations used in this example are restricted to singly reinforced rectangular cross sections (or sections that exhibit rectangular section behavior) with reinforcement in a single layer.

5.2. BEAM REINFORCEMENT DESIGN: SHEAR

Determine the required size and spacing of vertical U-stirrups for an 18 ft span, simply supported normal weight reinforced concrete beam.

Assume:
Interior exposure conditions
b_w = 12 in.
d = 19.5 in.
f_c' = 4000 psi
w_u = 4.82 kip/ft (includes self-weight)
ρ_f = 0.0270 (longitudinal reinforcement)

4EQ MST bar properties:
d_b = 0.45 in.
$A_{f,bar}$ = 0.16 in^2
E_f = 9427 ksi

Design material properties:
C_E = 0.8 (interior exposure)
$f_{fu} = C_E f_{fu}^*$ = (0.8) (181.1) = 144.9 ksi
r_b/d_b = 3

For the purposes of this example, the live load will be assumed to be present on the full span so that the design shear at the centerline of span is zero. (A design shear greater than zero at midspan is obtained by considering partial live loading of the span.)

1. Determine factored shear forces.

   At support: $V_u = 4.82\left(\frac{18}{2}\right) = 43.38\ kip$

   At distance d from support: $V_u = 43.38 - 4.82\left(\frac{19.5}{12}\right) = 35.55\ kip$

2. Determine shear strength provided by concrete.

$$\emptyset V_c = \emptyset\left(\frac{5}{2}k\right)2\sqrt{f_c'}b_w d$$

$$E_c = 57{,}000\sqrt{f_c'} = 57{,}000\sqrt{4000} = 3605\ ksi$$

$$n_f = \frac{E_f}{E_c} = \frac{9427}{3605} = 2.62$$

$$\rho_f n_f = (0.027)(2.62) = 0.0706$$

$$k = \sqrt{2\rho_f n_f + \left(\rho_f n_f\right)^2} - \rho_f n_f = \sqrt{2(0.0706) + (0.0706)^2} - (0.0706) = 0.3117$$

$$\emptyset V_c = (0.75)\left(\frac{5}{2}(0.3117)\right)2\frac{\sqrt{4000}}{1000}(12)(19.5) = 17.3\ kip$$

$$V_u = 35.55\ kips\ > \emptyset\frac{V_c}{2} = 8.65\ kip$$

   Therefore, shear reinforcement is required.

3. Compute $V_u - \phi V_c$ at critical section.

$$V_u - \emptyset V_c = 35.5 - 17.3 = 18.2\ kip\ < \emptyset 8\sqrt{f_c'}b_w d = 88.8\ kip \quad \text{OK}$$

4. Determine distance x_c from support beyond which shear reinforcement is not required for strength ($V_u = \phi V_c$):

$$x_c = \frac{V_u support - \emptyset V_c}{w_u} = \frac{(43.38) - (17.3)}{(4.82)} = 5.51\ ft$$

   Determine distance x_m from support beyond which shear reinforcement is not required ($Vu = \varphi Vc/2$):

$$x_c = \frac{V_u support - \emptyset V_c/2}{w_u} = \frac{(43.38) - (8.65)}{(4.82)} = 7.20\ ft$$

   Therefore, only minimum shear reinforcement is required between 5.51 ft and 7.20 ft from the supports. Shear reinforcement is not required past 7.20 ft from the supports.

5. Determine design tensile stress in shear reinforcement.

Tensile strength of bent bars:

$$f_{fb} = \left(0.05\frac{r_b}{d_b} + 0.3\right) f_{fu} \leq f_{fu}$$

$$f_{fb} = (0.05(3) + 0.3)(144.9) = 65.2\ ksi\ \leq 144.9\ ksi \quad \text{OK}$$

The design tensile strength is based on a strain of 0.004:

$$f_{fv} = 0.004E_f \leq f_{fb}$$

$$f_{fv} = 0.004(9427) = 37.7\ ksi \leq 81.5\ ksi \quad \text{OK}$$

6. Determine required spacing of vertical U-stirrups at the critical section.

$$\frac{A_{fv}}{s} = \frac{(V_u - \emptyset V_c)}{\emptyset f_{fv} d} = \frac{(35.55 - 17.3)}{(0.75)(37.7)(19.5)} = 0.0496\ {in^2}/{in.}$$

Assuming 4EQ MST U-stirrups (A_{fv} = 0.32in.^{2})

$$s = \frac{0.32}{0.0496} = 6.5\ in.$$

Check maximum permissible spacing of stirrups:

s = d/2 = 9.75 in. ≤ 24 in. because $V_u - \emptyset V_c = 18.2\ kip\ < \emptyset 4\sqrt{f_c{}'} b_w d = 44.4\ kip$ OK

Maximum stirrup spacing based on minimum shear reinforcement:

$$s = \frac{A_{fv,min} f_{fv}}{50 b_w} = \frac{(0.32)(37.7)(1000)}{50(12)} = 20.1\ in$$

Therefore, the spacing at the critical section is restricted to the smallest of 6.5, 9.75, and 20.1 in. Select spacing of 6 in. at the critical section.

Determine the distance x where a transition can be made to a spacing of 9 in. (to satisfy strength and maximum spacing requirement calculated above as d/2 = 9.75 in.):

$$\frac{A_{fv}}{s} = \frac{(V_u - \emptyset V_c)}{\emptyset f_{fv} d}$$

$$\frac{0.32}{9} = \frac{(V_u - \emptyset V_c)}{(0.75)(37.7)(19.5)}$$

$$(V_u - \emptyset V_c) = 19.6\ kip$$

$$V_u = 19.6 + 17.3 = 36.9\ kip$$

$$x = \frac{V_u support - V_u}{w_u} = \frac{(43.38) - (36.9)}{(4.82)} = 1.34\ ft$$

Therefore, a transition may be made from 6 to 9 in. spacing at 1.3 ft from the support.

7. Select a stirrup spacing arrangement that satisfies previous calculations.

   Place first stirrup at 2 in. from support. Use 6 in. spacing to 1 ft-8 in. from support. Use 9 in. spacing to 7 ft-8 in. from support.

   Note: Many designers would carry the reinforcement spacing of 9 in. through midspan, even though it is not required by calculations.

5.3. SLAB REINFORCMENT DESIGN: ONE WAY

Determine the required thickness and reinforcement for a one-way slab continuous over three or more equal spans. Center-to-center span l = 15 ft and clear span l_n = 14 ft.

Assume:
Interior exposure conditions
f_c' = 4000 psi
r_b/d_b = 3

4EQ MST bar properties:
d_b = 0.45 in.
$A_{f,bar}$ = 0.16 in^2
E_f = 9427 ksi

Design material properties:
C_E = 0.8 (interior exposure)
$f_{fu} = C_E f_{fu}^* = (0.8)(181.1) = 144.9$ ksi

Service loads:
w_D = slab self-weight (no superimposed dead load)
w_L = 50 lb/ft^2

1. Determine required slab thickness.
   Based on minimum thickness table, consider estimated depth. End span will control thickness:
   $h \approx \frac{l}{17} = \frac{(15)(12)}{17} = 10.6\ in.$ ACI 440.1R, Table 7.3.2.1

   Table is only intended to provide guidance for initial design, therefore, assume h = 10 in.
2. Compute the design moments using approximate moment analysis. Design will be based on the end span because it will yield the highest moments.

   Assume the end of the end span is integral with the support.
   10 in. slab weighs: [(10)/ (12)] (145) = 121 lb/ft^2
   Factored load: q_u = 1.2(121) + 1.6(50) = 225 lb/ft^2

   Positive moment at discontinuous end integral with support:

$+M_u = \frac{q_u l_n{}^2}{14} = \frac{(0.225)(14)^2}{14} = 3.15\ ft - kip/ft$ 318-14, Sec 8.3.3

Negative moment at exterior face of first interior support:

$-M_u = \frac{q_u l_n{}^2}{10} = \frac{(0.225)(14)^2}{10} = 4.41\ ft - kip/ft$ 318-14, Sec 8.3.3

3. Determine required reinforcement and select bars.

Assume section is tension-controlled. For this case, $f_f = f_{fu} = 144.9$ ksi and ϕ= 0.55. For interior exposure, clear cover is 0.75 in.

4EQ MST bar for flexural reinforcement.

$$d = 10 - \left[(0.75) + \frac{0.45}{2}\right] = 9.03\ in.$$

$$c_b = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fu}}\right) d = \left(\frac{0.003}{0.003+\frac{144.9}{9427}}\right)(9.03) = 1.48 \text{ in}$$

Use moment strength equation to solve for area of reinforcement.
Consider –M because it governs.

$$M_u = \emptyset M_{n,reqd} = \emptyset A_{f,reqd} f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right)$$

$$A_{f,reqd} = \frac{M_u}{\emptyset f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right)} = \frac{(4.41)(12)}{(0.55)(144.9)\left(9.03 - \frac{(0.85)(1.48)}{2}\right)} = 0.079\ in^2/ft$$

Note that this requirement is well less than the minimum reinforcement, as computed in the following.

$$\rho_{f,ts} = 0.0014 \le 0.0018\frac{60{,}000}{f_{fu}}\frac{E_s}{E_f} \le 0.0036$$

$$\rho_{f,ts} = 0.0014 \le 0.0018\frac{60{,}000}{144{,}900}\frac{29{,}000}{9427} \le 0.0036$$

$\rho_{f,ts} = 0.0014 \le 0.0023 \le 0.0036$, so $\rho_{f,ts} = 0.0036$

$$A_{f,min} = \rho_{f,min} bh = 0.0036(12)(10) = 0.43\ in^2/ft$$

Select 4EQ MST at 4 in. spacing ($A_f = 0.48\ in^2/ft$)

Verify assumption of tension-controlled behavior:

$$\rho_{fb} = 0.85\beta_1\frac{f_c{}'}{f_{fu}}\frac{E_f\varepsilon_{cu}}{E_f\varepsilon_{cu} + f_{fu}}$$

$E_f\varepsilon_{cu} = 9427\ ksi(0.003) = 28.3$ ksi

$$\rho_{fb} = 0.85(0.85)\frac{(4)}{(144.9)}\frac{(28.3)}{(28.3 + 144.9)} = 0.0033$$

$$\rho_f = \frac{A_f}{bd} = \frac{(0.48)}{(12)(9.03)} = 0.0044$$

$\frac{\rho_f}{\rho_{fb}} = \frac{0.0044}{0.0033} = 1.34 < 1.4$ OK – Tension controlled section.

The slab may be designed to be 10 in. thick with 4EQ MST at 4 in. spacing for –M. By observation, the same minimum reinforcement will be required for +M.

In addition to flexural strength, the slab should be examined for shear and the serviceability criteria of crack control, deflections, and creep rupture stress limits. Calculations related to these requirements are covered in other example problems in this chapter.

Further calculations show that this slab will be uncracked at service by a significant margin and is even uncracked at ultimate conditions. The slab will work but will be highly inefficient. Therefore, consider a more efficient slab design that is selected to be cracked at service.

4. Redesign the slab to be cracked at service loads.

When cracked, FRP-reinforced concrete slabs are seldom governed by flexural strength. Whereas a slab designed for flexural strength alone would have a ratio of service level moment to nominal moment strength (M_{serv}/M_n) of approximately 0.40 to 0.45, depending on the ratio of dead-to-live load, most FRP-reinforced slabs are governed by serviceability requirements and will exhibit ratios closer to 0.20 to 0.25. As a design approximation, design for a flexural strength corresponding to approximately twice the actual factored moment ($2.0M_u$).

Select a reinforcement ratio corresponding to a compression-controlled section, as this will promote the use of enough reinforcement to control cracking and reduce deflections.

Design for $2.0M_u$ and as a starting point, assume $\rho_f = 1.5\rho_{fb}$.

$\rho_f = 1.5\rho_{fb} = 1.5(0.0033) = 0.0050$

Calculate the stress in the tensile reinforcement (f_f) at ultimate conditions for the assumed value of ρ_f.

$$f_f = \sqrt{\frac{\left(E_f \varepsilon_{cu}\right)^2}{4} + \frac{0.85\beta_1 f_c'}{\rho_f} E_f \varepsilon_{cu}} - 0.5E_f \varepsilon_{cu} \leq f_{fu}$$

$$f_f = \sqrt{\frac{(28.3)^2}{4} + \frac{0.85(0.85)(4)}{(0.0050)}(28.3)} - 0.5(28.3) \leq 144.9\ ksi$$

$$f_f = 115.97\ ksi$$

Estimate dead load based on an assumed 6 in. slab thickness:

6 in. slab weighs = [(6)/ (12)] (145) = 73 lb/ft^2
Factored load = q_u = 1.2(73) + 1.6(50) = 168 lb/ft^2

Negative moment at exterior face of first interior support (governs):

$$-M_u = \frac{q_u {l_n}^2}{10} = \frac{(0.168)(14)^2}{10} = 3.30\ ft - kip/ft$$ 318-14, Sec 8.3.3

Use the moment capacity equation to determine a depth for the slab.

$$M_u = \emptyset M_n = \emptyset \rho_f f_f \left(1 - 0.59 \frac{\rho_f f_f}{f_c{'}}\right) bd^2$$

$$(2)(3.30)(12) = (0.65)(0.0050)(115.97)\left(1 - 0.59 \frac{(0.0050)(115.97)}{(4)}\right)(12)d^2$$

$$d = 4.38\ in$$

Assume 4EQ bars for flexural reinforcement.

$$h = (4.38) + (0.75) + \frac{(0.45)}{2} = 5.36\ in$$

Round up to be conservative (6 in. thick slab [h = 6 in.]).

The flexural capacity could be checked, but it should be satisfactory by inspection because the design was based on providing a capacity of 2.0M_u. Thus, capacity calculations are not necessary.

Select reinforcement for the slab. Assume the same reinforcement for –M and +M.

$$A_f = \rho_f bh = 0.0050(12)(6) = 0.36\ in^2/ft$$

Select 4EQ MST at 4 in. spacing (A_f = 0.48 in^2/ft)

5. Select temperature and shrinkage reinforcement for transverse direction.

$$\rho_{f,ts} = 0.0014 \leq 0.0018 \frac{60{,}000}{f_{fu}} \frac{E_s}{E_f} \leq 0.0036$$

$$\rho_{f,ts} = 0.0014 \leq 0.0018 \frac{60{,}000}{144{,}900} \frac{29{,}000}{9427} \leq 0.0036$$

$\rho_{f,ts} = 0.0014 \leq 0.0023 \leq 0.0036$, so $\rho_{f,ts}$ = 0.0036

$$A_{f,min} = \rho_{f,min} bh = 0.0036(12)(6) = 0.26\ in^2/ft$$

Select 4EQ MST at 6 in. spacing (A_f = 0.32 in^2/ft)

Note: The slab should now be examined for shear, crack control, deflections, and creep rupture. Calculations related to these requirements are covered in other example problems in this chapter. If these design criteria are not satisfied, then the slab thickness may be increased incrementally, or additional reinforcement added.

5.4. FOUNDATION REINFORCMENT DESIGN: SHALLOW
Determine the reinforcement for 5 ft square footing. Column dimensions = 12” x 12”.

Assume:
Exterior exposure conditions
Footing thickness = 1’-6”
Clear Cover = 3”
f_c' = 3000 psi
Pu = 60 kip
q_s = 2.5 ksf

4EQ MST bar properties:
d_b = 0.45 in.
$A_{f,bar}$ = 0.16 in^2
E_f = 9427 ksi

Design material properties:
C_E = 0.7 (exterior exposure)
$f_{fu} = C_E f_{fu}^*$ = (0.7) (181.1) = 126.8 ksi

1. Determine moment at critical section at face of column.

$$M_u = q_s l_w \frac{a^2}{2}$$

$$a = \frac{b_{w,footing} - b_{w,column}}{2} = \frac{5-1}{2} = 2\ ft$$

$$M_u = (2.5)(5)\frac{(2^2)}{2} = 25\ ft - kip$$

2. Compute required, As assuming tension-controlled section (ϕ=0.55)

Assume section is tension-controlled.
4EQ MST bar for flexural reinforcement.

$$d = 18 - \left[(3) + \frac{0.45}{2}\right] = 14.78\ in.$$

$$c_b = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{fu}}\right) d = \left(\frac{0.003}{0.003+\frac{126.8}{9427}}\right)(14.78) = 2.69 \text{ in}$$

$$M_u = \emptyset M_{n,reqd} = \emptyset A_{f,reqd} f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right)$$

$$A_{f,reqd} = \frac{M_u}{\emptyset f_{fu}\left(d - \frac{\beta_1 c_b}{2}\right)} = \frac{(25)(12)}{(0.55)(126.8)\left(14.78 - \frac{(0.85)(2.69)}{2}\right)} = 0.316\ in^2$$

Check requirement for minimum reinforcement.

$$\rho_{f,ts} = 0.0014 \leq 0.0018\frac{60{,}000}{f_{fu}}\frac{E_s}{E_f} \leq 0.0036$$

$$\rho_{f,ts} = 0.0014 \leq 0.0018\frac{60{,}000}{126800}\frac{29{,}000}{9427} \leq 0.0036$$

$\rho_{f,ts} = 0.0014 \leq 0.0026 \leq 0.0036$, so $\rho_{f,ts}$ = 0.0026

$$A_{f,min} = \rho_{f,min}bh = 0.0026(60)(14.78) = 2.3\ in^2$$

3. Determine required reinforcement and select bars.

   (3) 4EQ MST (As = 0.48 in^2) meets the tension-controlled section.

   (15) 4EQ MST (As =2.4 in^2) required for minimum reinforcement.

   Use 4EQ MST @ 4” o.c. each way

4. Check the development for reinforcement.

   $l_d = \frac{d_b\left(\frac{f_{fr}}{\sqrt{f_c'}} - 340\right)\psi_t}{13.6 + \frac{c_b}{d_b}} d_b$ where $\frac{c_b}{d_b} \leq 3.5$

   Ψt =1.0

   $l_d = \frac{(0.45)\left(\frac{(126800)}{\sqrt{3}000} - 340\right)(1.0)}{13.6 + 3.5}(0.45)$ =23.4 in

   $\frac{60}{2} - \left(\frac{12}{2}\right) = 24\ in\ \geq 23.4\ in$ OK